## Assignment 5.

1. Find the maximum value of the function $y = x^2 e^{1-2x}$ for $x > 0$. [4]

2. Find the equation of the tangent to the curve of $y = x^2 \ln x$ at $x = e$. [5]

3. Find the equation of the normal to the curve of $y = x \tan 2x$ at the point where $x = \frac{1}{2}\pi$. [5]

4. The curve $y = \dfrac{e^x}{\cos x}$, for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, has one stationary point. Find the $x$-coordinate of this point. [5]

5. For a two-dimensional curve written in the form $y = f(x)$, the **curvature** of the curve at a particular point is defined as

$$\kappa = \frac{\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}}{\left[1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{\frac{3}{2}}}.$$

(a) Find the curvature of $y = \mathrm{e}^x$ at the point $(0, 1)$. [3]

(b) Find the curvature of $y = \ln x$ at the point $(1, 0)$. [3]

(c) Find the curvature of $y = \arctan x$ at the point $\left(1, \frac{\pi}{4}\right)$. [5]

6. (†) Differentiate the function $y = x^x$. [4]

**Total mark** of this assignment: $30 + 4$.

The symbol (†) indicates a bonus question. Finish other questions before working on this one.